

- 1 Find the gradient of the straight line with equation $5x + 2y = 7$

Equation of straight line : $y = mx + c$

Rearrange equation to
 $y = mx + c$

where $m = \text{gradient}$
 $c = y\text{-intercept}$

$$5x + 2y = 7$$

$$2y = -5x + 7$$

$$y = \boxed{-\frac{5}{2}}x + \frac{7}{2} \quad \textcircled{1}$$

← gradient, m

$$-\frac{5}{2} \quad \textcircled{1}$$

(Total for Question 1 is 2 marks)

2 P and Q are two points.

The coordinates of P are $(-1, 6)$

The coordinates of Q are $(5, -4)$

↙ midpoint of PQ

Find an equation of the perpendicular bisector of PQ .

Give your answer in the form $ax + by + c = 0$ where a , b and c are integers.

Finding midpoint of PQ :

$$\left(\frac{-1+5}{2}, \frac{6+(-4)}{2} \right) = (2, 1) \quad (1)$$

Finding gradient of line PQ :

$$m = \frac{(-4-6)}{(5-(-1))} = \frac{-10}{6} = \frac{-5}{3} \quad (1)$$

Finding gradient of perpendicular bisector :

$$m_{PB} = - \frac{1}{\frac{-5}{3}} = \frac{3}{5} \quad (1)$$

Finding equation of perpendicular bisector :

known value : point $(2, 1)$ and gradient $= \frac{3}{5}$

$$y = mx + c$$

$$1 = \frac{3}{5}(2) + c$$

$$c = 1 - \frac{6}{5}$$

$$= -\frac{1}{5} \quad (1)$$

$$\text{Equation} = y = \frac{3}{5}x - \frac{1}{5} \quad (1)$$

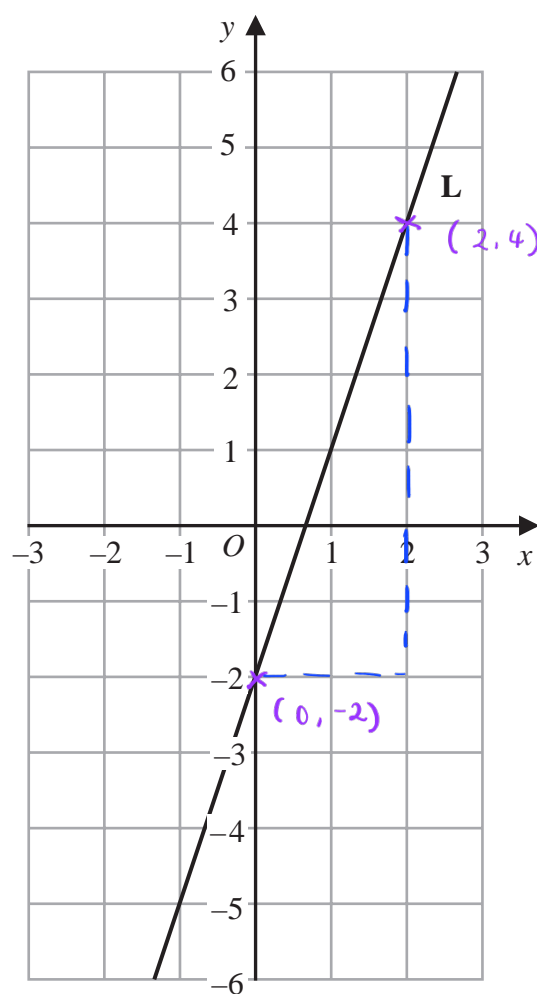
$$\therefore 5y = 3x - 1$$

$$\therefore 3x - 5y - 1 = 0$$

$$3x - 5y - 1 = 0 \quad (1)$$

(Total for Question 2 is 6 marks)

3 The line **L** is shown on the grid.



Find an equation for **L**.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y\text{-intercept} = -2$$

$$m = \frac{4 - (-2)}{2 - 0} = \frac{6}{2} = 3 \quad (1)$$

$$y = mx + c$$

$$y = 3x - 2 \quad (1)$$

$$y = 3x - 2$$

(Total for Question 3 is 2 marks)

4 L_1 and L_2 are two straight lines.

The origin of the coordinate axes is O .

L_1 has equation $5x + 10y = 8$

L_2 is perpendicular to L_1 and passes through the point with coordinates $(8, 6)$

L_2 crosses the x -axis at the point A .

L_2 intersects the straight line with equation $x = -3$ at the point B .

Find the area of triangle AOB .

Show your working clearly.

$$\text{Equation of } L_1 : 5x + 10y = 8$$

$$10y = -5x + 8$$

$$y = -\frac{1}{2}x + \frac{4}{5}$$

in terms of $y = mx + c$

$$\text{Gradient of } L_1 : -\frac{1}{2}$$

$$\text{Gradient of } L_2 : 2$$

①

$$m_{L_1} = -\frac{1}{m_{L_2}}$$

$$\text{Equation of } L_2 : 6 = 2(8) + c$$

$$c = -10$$

$$\therefore y = 2x - 10 \quad \text{①}$$

when L_2 crosses point A :

$$y = 0 : 0 = 2x - 10$$

$$x = 5 \quad \text{①}$$

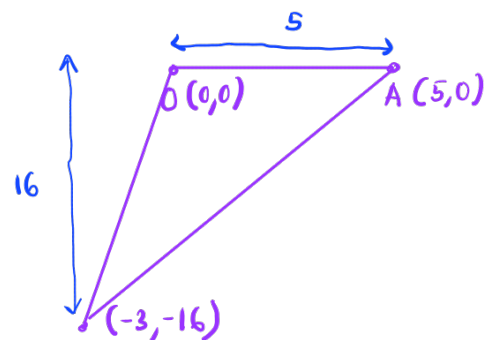
$\therefore L_2$ crosses point A at $(5, 0)$

when L_2 intersects at point B :

$$x = -3 : y = 2(-3) - 10$$

$$y = -16$$

$\therefore L_2$ intersects at $(-3, -16)$



$$\text{Area} = \frac{1}{2} \times 5 \times 16 \quad \text{①}$$

$$= 40 \quad \text{①}$$

40

(Total for Question 4 is 5 marks)

Line **L** has equation $y = 2 - 3x$

5 (b) Write down the gradient of line **L**.

$$y = \underline{-3}x + 2$$

↑
m

-3 ①

(1)

(Total for Question 5 is 1 marks)

6 Triangle HJK is isosceles with $HJ = HK$ and $JK = \sqrt{80}$

H is the point with coordinates $(-4, 1)$

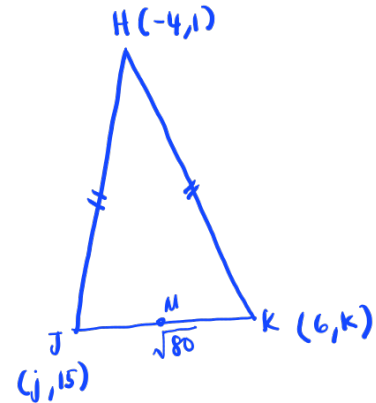
J is the point with coordinates $(j, 15)$ where $j < 0$

K is the point with coordinates $(6, k)$

M is the midpoint of JK .

The gradient of HM is 2

Find the value of j and the value of k .



Given : gradient of $HM = 2$

$$\text{gradient of } JK = \frac{-1}{2} = -\frac{1}{2} \quad (1)$$

$$-\frac{1}{2} = \frac{(k-15)}{(6-j)}$$

$$-6+j = 2k-30$$

$$j = 2k-24 \quad (1)$$

Given : length of $JK = \sqrt{80}$

$$\sqrt{(6-j)^2 + (k-15)^2} = \sqrt{80}$$

$$(6-j)^2 + (k-15)^2 = 80 \quad (1)$$

$$j^2 - 12j + 36 + k^2 - 30k + 225 = 80$$

$$j^2 - 12j + k^2 - 30k = -181 \quad (2)$$

substitute (1) into (2) :

$$(2k-24)^2 - 12(2k-24) + k^2 - 30k = -181$$

$$4k^2 - 96k + 576 - 24k + 288 + k^2 - 30k = -181$$

$$5k^2 - 150k + 1045 = 0 \quad (1)$$

$$k = \frac{150 \pm \sqrt{(-150)^2 - 4(5)(1045)}}{2(5)} \quad (1)$$

$$= \frac{150 \pm \sqrt{1600}}{10}$$

$$= \frac{150 \pm 40}{10}$$

$$k = 19 \text{ or } 11$$

substitute k values into ①

$$j = 2(19) - 24 \quad \text{or} \quad j = 2(11) - 24$$
$$= 14 \quad \text{or} \quad j = -2$$

since $j < 0$,

$$\therefore j = -2 \text{ and } k = 11 \quad \text{①}$$

$$j = \overset{-2}{\dots\dots\dots}$$

$$k = \overset{11}{\dots\dots\dots}$$

(Total for Question 6 is 6 marks)

- 7 (a) Write down an equation of a line that is parallel to the line with equation $y = 7 - 4x$

$$y = -4x \quad (1)$$

The line **L** passes through the points with coordinates $(-3, 1)$ and $(2, -2)$

- (b) Find an equation of the line that is perpendicular to **L** and passes through the point with coordinates $(-6, 4)$

Give your answer in the form $ax + by + c = 0$ where a , b and c are integers.

$$\text{gradient of L : } \frac{-2-1}{2-(-3)} = -\frac{3}{5} \quad (1)$$

$$\text{gradient of line } \perp \text{ to L : } \frac{-1}{-\frac{3}{5}} = \frac{5}{3} \quad (1)$$

$$\text{Equation of line } \perp \text{ to L : } 4 = \frac{5}{3}(-6) + c$$

$$c = 14 \quad (1)$$

$$\therefore y = \frac{5}{3}x + 14$$

$$3y = 5x + 42$$

$$5x - 3y + 42 = 0 \quad (1)$$

$$5x - 3y + 42 = 0$$

(4)

(Total for Question 7 is 5 marks)

8 The straight line **L** passes through the points (4, -1) and (6, 4)

The straight line **M** is perpendicular to **L** and intersects the y-axis at the point (0, 8)

Find the coordinates of the point where **M** intersects the x-axis.

$$\text{gradient of line L} : \frac{4 - (-1)}{6 - 4}$$

$$= \frac{5}{2} \quad (1)$$

$$\text{gradient of line M} = \frac{-1}{m_L}$$

$$= \frac{-1}{\frac{5}{2}} = -\frac{2}{5} \quad (1)$$

$$\text{Equation of line M} : y = -\frac{2}{5}x + 8 \quad \leftarrow \text{intersects at y-axis (0,8)}$$

when M intersects x-axis, $y = 0$

$$0 = -\frac{2}{5}x + 8$$

$$\frac{2}{5}x = 8$$

$$x = \frac{8 \times 5}{2}$$

$$= 20 \quad (1)$$

\therefore M intersects x-axis at (20, 0) (1)

(20 , 0)

(Total for Question 8 is 4 marks)

9 ABC is an isosceles triangle with $AB = AC$.

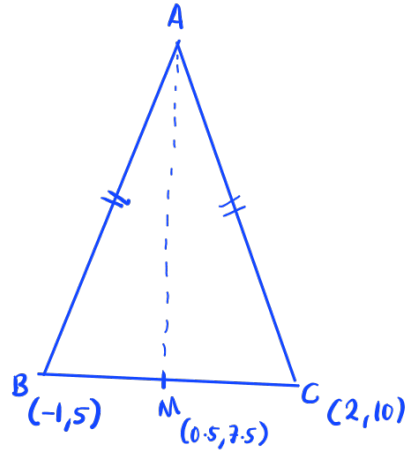
B is the point with coordinates $(-1, 5)$

C is the point with coordinates $(2, 10)$

M is the midpoint of BC .

Find an equation of the line through the points A and M .

Give your answer in the form $py + qx = r$ where p , q and r are integers.



$$\begin{aligned}\text{midpoint of } BC &= \left(\frac{2+(-1)}{2}, \frac{10+5}{2} \right) \\ &= (0.5, 7.5) \quad (1)\end{aligned}$$

$$\begin{aligned}\text{gradient of line } BC &: \frac{10-5}{2-(-1)} \\ &= \frac{5}{3} \quad (1)\end{aligned}$$

$$\begin{aligned}\text{gradient of line } MA &= \frac{-1}{m_{BC}} \\ &= -\frac{3}{5} \quad (1)\end{aligned}$$

$$\text{Equation of line } MA = 7.5 = -\frac{3}{5}(0.5) + c$$

$$\begin{aligned}c &= 7.5 + 0.3 \\ &= \frac{39}{5} \quad (1)\end{aligned}$$

$$y = -\frac{3}{5}x + \frac{39}{5}$$

$$5y = -3x + 39$$

$$5y + 3x = 39 \quad (1)$$

$$5y + 3x = 3q$$

(Total for Question 9 is 5 marks)

- 10** Point A has coordinates $(5, 8)$
Point B has coordinates $(9, -4)$

(a) Work out the gradient of AB .

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{gradient} = \frac{8 - (-4)}{5 - 9}$$

$$= \frac{12}{-4} \text{ (1)}$$

$$= -3 \text{ (1)}$$

$$\frac{-3}{(2)}$$

(Total for Question 10 is 2 marks)

11 A rectangle $ABCD$ is to be drawn on a centimetre grid such that

A has coordinates $(-4, -2)$

B has coordinates $(1, 10)$

C has coordinates $(19, a)$

D has coordinates (b, c)

(a) Work out the value of a , the value of b and the value of c .

$$\text{Difference in } x\text{-axis between } AB = 1 - (-4) = 5$$

$$\text{That means } b = 19 - 5$$

$$b = 14 \quad (1)$$

$$\begin{aligned} \text{Gradient } AB &= \frac{10 - (-2)}{1 - (-4)} \\ &= \frac{12}{5} \quad (1) \end{aligned}$$

$$\begin{aligned} \text{Gradient } BC &= \frac{a - 10}{19 - 1} \\ &= \frac{a - 10}{18} \end{aligned}$$

$$\frac{12}{5} \times \frac{a - 10}{18} = -1$$

$$\frac{12(a - 10)}{90} = -1$$

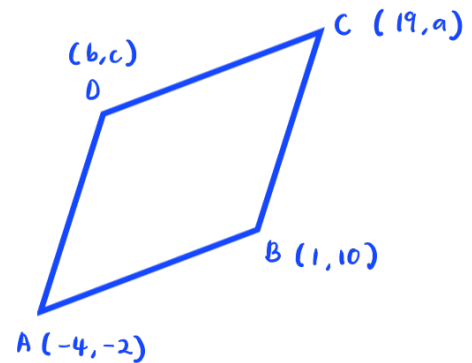
$$12a - 120 = -90$$

$$12a = 30$$

$$a = 2.5 \quad (1)$$

$$\begin{aligned} \text{Difference in } y\text{-axis between } AB &= 10 - (-2) \\ &= 12 \end{aligned}$$

$$c = 2.5 - 12 = -9.5 \quad (1)$$



perpendicular lines :

$$m_1 m_2 = -1$$

$$a = \dots\dots\dots 2.5$$

$$b = \dots\dots\dots 14$$

$$c = \dots\dots\dots -9.5$$

(4)

(b) Calculate the perimeter, in centimetres, of rectangle $ABCD$.

$$\begin{aligned} AB &= \sqrt{(1 - (-4))^2 + (10 - (-2))^2} \\ &= \sqrt{5^2 + 12^2} \\ &= 13 \text{ ①} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(19 - 1)^2 + (2.5 - 10)^2} \\ &= 19.5 \text{ ①} \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= 2(13) + 2(19.5) \\ &= 65 \text{ cm ①} \end{aligned}$$

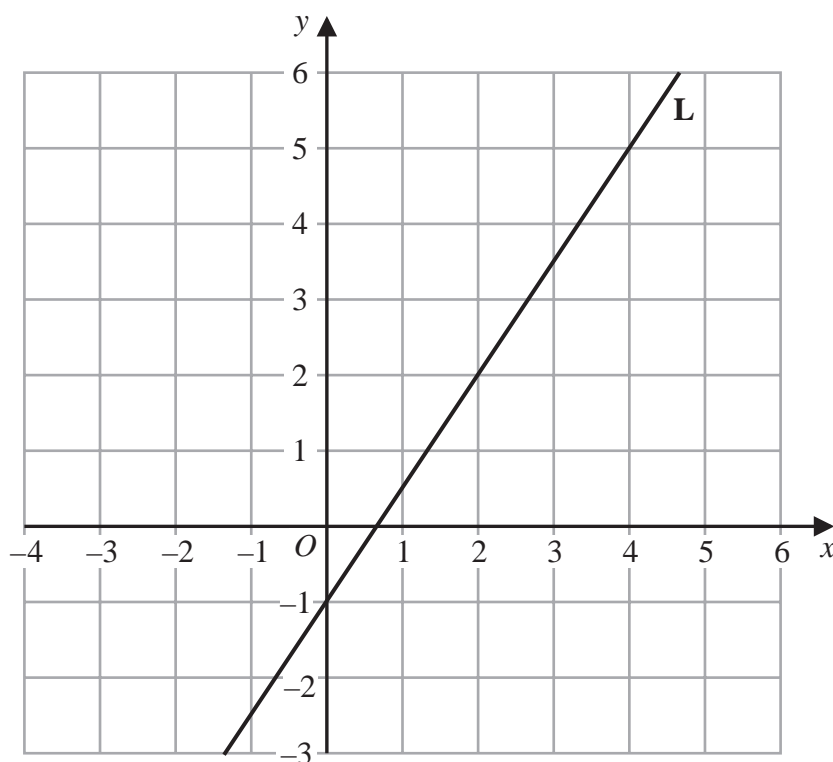
65

..... cm

(3)

(Total for Question 11 is 7 marks)

12 Line **L** is drawn on the grid.



Find an equation for **L**

Give your answer in the form $y = mx + c$

$$c = y\text{-intercept} = -1 \quad (1)$$

$$\text{gradient} : \frac{5 - (-1)}{4 - 0}$$

$$: \frac{6}{4} = \frac{3}{2} \quad (1)$$

$$y = \frac{3}{2}x - 1 \quad (1) \quad - y = mx + c$$

$$y = \frac{3}{2}x - 1$$

(Total for Question 12 is 3 marks)

- 13 $ABCD$ is a kite, with diagonals AC and BD , drawn on a centimetre square grid, with a scale of 1 cm for 1 unit on each axis.

A is the point with coordinates $(-3, 4)$

The diagonals of the kite intersect at the point M with coordinates $(0, 2)$

Given that $AB = AD = 6.5$ cm and the x coordinate of B is positive,

find the coordinates of the points B and D .

$$m_{AM} = \frac{4-2}{-3} = -\frac{2}{3} \quad (1)$$

$$m_{BD} = \frac{3}{2}$$

$$\text{Equation of line } BD: y-2 = \frac{3}{2}x$$

$$y = \frac{3}{2}x + 2 \quad (1)$$

$$AM = \sqrt{(-3-0)^2 + (4-2)^2} = \sqrt{13}$$

$$BM = \sqrt{(x-0)^2 + (y-2)^2} = \sqrt{x^2 + (y-2)^2} \quad (1)$$

$$AB^2 = AM^2 + BM^2$$

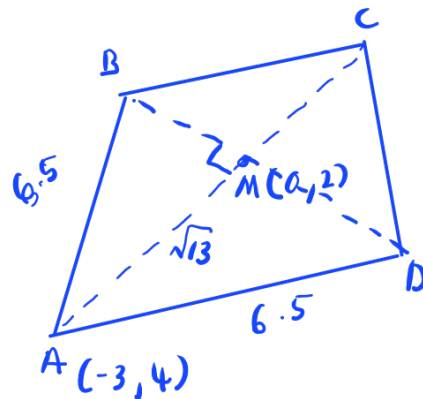
$$(6.5)^2 = 13 + x^2 + (y-2)^2 \quad (1)$$

$$\frac{117}{4} = x^2 + (y-2)^2$$

$$\frac{117}{4} = x^2 + \left(\frac{3}{2}x\right)^2 \quad (1)$$

$$\frac{117}{4} = \frac{13}{4}x^2 \quad (1)$$

$$x^2 = \frac{117}{13} = 9$$



$$x = \pm 3$$

$$x = 3, y = 6.5$$

$$x = -3, y = -2.5$$

①

(.....³.....,.....^{6.5}.....)

(.....⁻³.....,.....^{-2.5}.....)

(Total for Question 13 is 7 marks)

14 ABC is a triangle in which $\angle ABC = 90^\circ$

p and q are integers such that

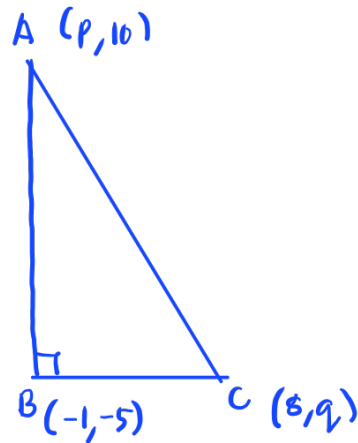
the coordinates of A are $(p, 10)$

the coordinates of B are $(-1, -5)$

the coordinates of C are $(8, q)$

Given that the gradient of AC is $-\frac{6}{7}$

work out the value of p and the value of q



$$\text{gradient } AB : \frac{10 - (-5)}{p - (-1)} = \frac{15}{p+1} \quad (1)$$

$$\text{gradient } BC : \frac{q - (-5)}{8 - (-1)} = \frac{q+5}{9}$$

$$\frac{15}{p+1} \times \frac{q+5}{9} = -1 \quad (1)$$

$$5q + 25 = -3p - 3 \quad (1)$$

$$5q + 3p = -28 \quad (1)$$

$$p = \frac{-28 - 5q}{3}$$

$$\text{gradient } AC : \frac{10 - q}{p - 8} = -\frac{6}{7}$$

$$70 - 7q = -6p + 48$$

$$6p - 7q = -22 \quad (2)$$

$$6 \left(\frac{-28 - 5q}{3} \right) - 7q = -22 \quad (1)$$

$$-56 - 10q - 7q = -22$$

$$-17q = -22 + 56$$

$$-17q = 34$$

$$q = -2$$

$$p = \frac{-28 - 5(-2)}{3}$$

$$= \frac{-28 + 10}{3}$$

$$= \frac{-18}{3}$$

$$= -6$$

$$p = \dots \dots \dots -6 \quad \textcircled{1} \dots \dots \dots$$

$$q = \dots \dots \dots -2 \dots \dots \dots$$

(Total for Question 14 is 5 marks)

15 The straight line **L** has equation $2y + 7x = 10$

(a) Find the gradient of **L**

$$2y = -7x + 10 \quad (1)$$

$$y = -\frac{7}{2}x + 5$$

$$-3.5 \quad (1)$$

(2)

(b) Find the coordinates of the point where **L** crosses the y-axis.

$$x = 0$$

$$y = -\frac{7}{2}(0) + 5$$

$$y = 5$$

$$(0, 5)$$

$$(\quad 0 \quad, \quad 5 \quad) \quad (1)$$

(Total for Question 15 is 3 marks)

- 16 G is the point on the curve with equation $y = 8x^2 - 14x - 6$ where the gradient is 10
The straight line Q passes through the point G and is perpendicular to the tangent at G

Find an equation for Q

Give your answer in the form $ax + by + c = 0$ where a , b and c are integers.

$$\text{gradient, } \frac{dy}{dx} = 16x - 14 \quad (1)$$

$$16x - 14 = 10 \quad (1)$$

$$x = \frac{24}{16} = 1.5$$

$$y = 8(1.5)^2 - 14(1.5) - 6$$

$$= -9 \quad (1)$$

$$G(1.5, -9)$$

$$m_Q = -\frac{1}{10}$$

$$-9 = -\frac{1}{10} \left(\frac{3}{2} \right) + c$$

$$-9 + \frac{3}{20} = c$$

$$\frac{-177}{20} = c \quad (1)$$

$$y = -\frac{1}{10}x - \frac{177}{20}$$

$$20y = -2x - 177$$

$$2x + 20y + 177 = 0 \quad (1)$$

$$2x + 20y + 177 = 0$$

(Total for Question 16 is 5 marks)

17 $ABCD$ is a trapezium with AB parallel to DC

A is the point with coordinates $(-4, 6)$

B is the point with coordinates $(2, 3)$

D is the point with coordinates $(-1, 8)$

The trapezium has one line of symmetry.

The line of symmetry intersects CD at the point E

Work out the coordinates of the point E

$$\begin{aligned}\text{midpoint } AB &: \left(\frac{-4+2}{2}, \frac{6+3}{2} \right) \\ &= (-1, 4.5) \quad (1)\end{aligned}$$

$$\text{gradient } AB : \frac{6-3}{-4-2} = -\frac{1}{2} \quad (1)$$

$$\text{gradient of symmetry line} = 2 \quad (1)$$

$$DC: \quad y - 8 = -0.5(x - (-1))$$

$$\begin{aligned}y - 8 &= -0.5x - 0.5 \\ y &= -0.5x + 7.5 \quad (1)\end{aligned}$$

$$\text{symmetry line: } y - 4.5 = 2(x - (-1))$$

$$y - 4.5 = 2x + 2$$

$$y = 2x + 6.5 \quad (2) \quad (1)$$

$$2x + 6.5 = -0.5x + 7.5$$

$$2.5x = 1$$

$$x = \frac{1}{2.5} = 0.4$$

$$y = 2(0.4) + 6.5 = 7.3$$

(1)

(0.4, 7.3)

(Total for Question 17 is 6 marks)

18 $ABCD$ is a kite.

$$AB = AD \text{ and } CB = CD$$

The point B has coordinates $(k, 1)$ where k is a negative constant.

The point D has coordinates $(8, 7)$

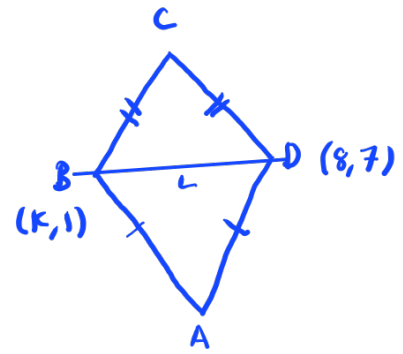
The straight line L passes through the points B and D

The straight line L is parallel to the line with equation $5y - 3x = 6$

Find an equation of AC

Give your answer in the form $px + qy = r$ where p , q and r are integers.

Show your working clearly.



$$\text{gradient of } L : 5y = 3x + 6$$

$$y = \frac{3}{5}x + \frac{6}{5} \quad (1)$$

$$m_L = \frac{3}{5}$$

$$\frac{3}{5} = \frac{7-1}{8-k}$$

$$24 - 3k = 35 - 5$$

$$3k = 24 - 30$$

$$3k = -6$$

$$k = -2 \quad (1)$$

$$\text{midpoint of } BD : \left(\frac{8+(-2)}{2}, \frac{7+1}{2} \right)$$

$$= (3, 4) \quad (1)$$

$$\text{gradient of } AC : -\frac{5}{3} \quad (1)$$

$$\text{Equation of } AC : y - 4 = -\frac{5}{3}(x - 3) \quad (1)$$

$$3y - 12 = -5x + 15$$

$$3y = -5x + 27$$

$$5x + 3y = 27 \quad (1)$$

$$5x + 3y = 27$$

(Total for Question 18 is 6 marks)

- 19 The straight line with equation $y - 2x = 7$ is the perpendicular bisector of the line AB where A is the point with coordinates $(j, 7)$ and B is the point with coordinates $(6, k)$

Find the coordinates of the midpoint of the line AB

Show clear algebraic working.

$$y = 2x + 7$$

$$m = 2$$

$$m_{AB} = -\frac{1}{2} \quad (1)$$

$$-\frac{1}{2} = \frac{k-7}{6-j} \quad (1)$$

$$-6+j = 2k-14$$

$$2k-j = 8 \quad (1)$$

$$\text{midpoint of } AB : \left(\frac{j+6}{2}, \frac{7+k}{2} \right) \quad (1)$$

$$\frac{7+k}{2} = 2\left(\frac{j+6}{2}\right) + 7$$

$$7+k = 2j+12+14 \quad (1)$$

$$k = 2j+19 \quad (2)$$

substitute (2) into (1) :

$$2(2j+19) - j = 8$$

$$4j+38-j = 8$$

$$3j = -30$$

$$j = -10 \quad (1)$$

$$\begin{aligned} k &= 2(-10) + 19 \\ &= -1 \end{aligned}$$

$$\begin{aligned}\text{midpoint of AB} &: \left(\frac{-10+6}{2}, \frac{7-1}{2} \right) \\ &= (-2, 3) \quad \textcircled{1}\end{aligned}$$

$$(\text{.....}^{-2}\text{.....}, \text{.....}^3\text{.....})$$

(Total for Question 19 is 6 marks)

20 A is the point with coordinates $(-5, 12)$

B is the point with coordinates $(19, -48)$

Find an equation of the straight line that passes through the points A and B

$$\text{gradient} = \frac{12 - (-48)}{-5 - 19} = \frac{60}{-24} = -2.5 \quad (1)$$

$$\text{equation: } 12 = -2.5(-5) + c$$

$$12 = 12.5 + c$$

$$c = -0.5 \quad (1)$$

$$y = -2.5x - 0.5 \quad (1)$$

$$y = -2.5x - 0.5$$

(Total for Question 20 is 3 marks)

21 $ABCD$ is a kite with $AB = AD$ and $CB = CD$

A is the point with coordinates $(-2, 10)$

B is the point with coordinates $\left(-\frac{27}{5}, 4\right)$

C is the point with coordinates $(4, -5)$

Work out the coordinates of D

$$\text{gradient } AC : \frac{-5-10}{4-(-2)} = \frac{-15}{6} = -\frac{5}{2} \quad (1)$$

$$\text{equation of } AC : 10 = -\frac{5}{2}(-2) + c$$

$$c = 10 - 5 = 5$$

$$\therefore y = -\frac{5}{2}x + 5 \quad (1)$$

$$\text{gradient } BD : \frac{2}{5}$$

$$\text{equation of } BD : 4 = \frac{2}{5}\left(-\frac{27}{5}\right) + c$$

$$4 = -\frac{54}{25} + c$$

$$c = \frac{154}{25}$$

$$\therefore y = \frac{2}{5}x + \frac{154}{25} \quad (1)$$

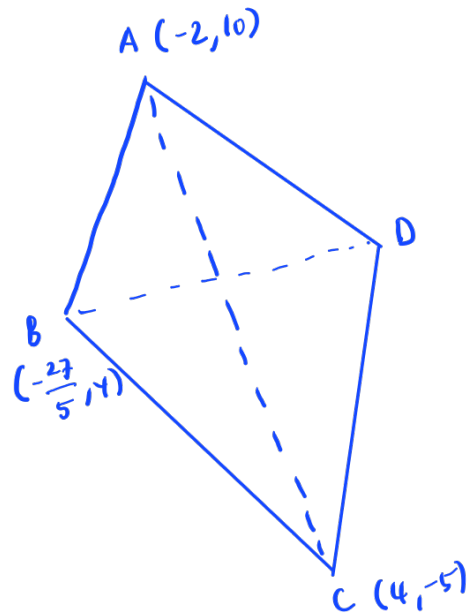
$$-\frac{5}{2}x + 5 = \frac{2}{5}x + \frac{154}{25} \quad (1)$$

$$\frac{2}{5}x + \frac{5}{2}x = 5 - \frac{154}{25}$$

$$2.9x = -\frac{29}{25} \quad (1)$$

$$x = -\frac{10}{25} = -\frac{2}{5}$$

$$y = -\frac{5}{2}\left(-\frac{2}{5}\right) + 5 = 6$$



intersection between AC and BD is $(-\frac{2}{5}, 6)$

$$\left(-\frac{2}{5}, 6\right) = \left(\frac{-\frac{27}{5} + x_D}{2}, \frac{4 + y_D}{2}\right)$$

$$x_D : \frac{-4}{5} + \frac{27}{5} = \frac{23}{5}$$

$$y_D : 12 - 4 = 8$$

①

$$\left(\frac{23}{5}, 8\right)$$

(Total for Question 21 is 6 marks)